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Since $\tan c = 1/\sqrt{e} = \cot b$, therefore, $c = 90° - b$. Hence, $a + b + c + f = 180$. But $a + b + c + d = 180°$. Hence, $d = f$.

Hence, DA and DE coincide and the ball does return to A.

Also solved by HORACE OLSON, A. M. HARDING, and HAROLD T. DAVIS.

NUMBER THEORY.

230. Proposed by E. B. ESCOTT, Ann Arbor, Michigan.

Find three numbers such that their sum, the sum of their squares, and the sum of their cubes shall be a cube.

SOLUTION BY E. T. BELL, Seattle, Washington.

Presumably the proposer wished three *integers*, (x, y, z) satisfying the conditions. No restrictions being imposed, we may assume one of the cubes to be zero, and choose $(x + y + z) = 0$, which simplifies the problem very considerably, reducing it in effect to the solution in integers (m, v) of

$$m^2 = 36v^3 + 1. \tag{1}$$

A solution (x, y, z) is:

$$\begin{array}{rrr}
x = 146, & x^2 = 21\,316, & x^3 = 3\,112\,136, \\
y = -1\,314, & y^2 = 1\,726\,596, & y^3 = -2\,268\,747\,144, \\
z = 1\,168, & z^2 = 1\,364\,224, & z^3 = 1\,593\,413\,632, \\
w = -876, & & w^3 = -672\,221\,376,
\end{array}$$

whence

$$x + y + z = 0^3,$$
$$x^2 + y^2 + z^2 = x^3,$$
$$x^3 + y^3 + z^3 = w^3 = (-6x)^3,$$

which may be most readily checked from the resolutions into prime factors:

$$x = 2 \times 73; \quad y = -2 \times 3^2 \times 73; \quad z = 2^4 \times 73; \quad w = -2^2 \times 3 \times 73;$$

clearly, (xt^3, yt^3, zt^3), where t is an arbitrary integer, is also an integral solution.

More generally, (m, v) being any solution of (1), and k arbitrary,

$$\begin{aligned}
x &= 18(m^2 + 3)(m + 1)^2(m - 1)^2k^3, \\
y &= -9(m^2 + 3)(m + 1)^3(m - 1)^2k^3, \\
z &= 9(m^2 + 3)(m + 1)^2(m - 1)^3k^3,
\end{aligned} \tag{2}$$

give

$$x + y + z = 0^3,$$
$$\begin{aligned}
x^2 + y^2 + z^2 &= \{2^33^4k^2v^4(m^2 + 3)\}^3 = \{2^53^4v^4k^2(3^2v^2 + 1)\}^3, \\
x^3 + y^3 + z^3 &= \{-2^53^7(m^2 + 3)v^7k^3\}^3 = \{-2^73^7v^7k^3(3^2v^2 + 1)\}^3,
\end{aligned} \tag{3}$$

which, for $k = 2^{-4}3^{-2}$, and the solution $(m, v) = (17, 2)$ of (1) gives the solution first stated.

Note.—The analysis for solution 2, which would take about two pages of the MONTHLY, can be written out if thought of sufficient interest, but it involves nothing new. Incidentally, the above solution carries with it that of many other curious indeterminate systems, when (3) is combined with Newton's formulæ for the sums of like powers of the roots of an algebraic equation; thus, we are shown how to find integers (a, b, c, p, r) satisfying

$$a^5 + b^5 + c^5 = 30p^3,$$
$$a^7 + b^7 + c^7 = 28r^3.$$

There is an infinite chain of such results, which are all more or less in the nature of accidents.